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Probabilistic initial failure strength of hybrid and non-hybrid laminates

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This paper deals with the initial failure of unidirectional hybrid laminates and $[\pm\theta/90]_s$ non-hybrid laminates. The initial failure strength or strain at the failure of low elongation layers is analysed by the statistical approach based upon the weakest link model. First ply failure is adopted as a criterion for the initial failure of the laminate. An expression for determining the first ply failure strength has been derived and this expression can be reduced to a volumetric relation as a special case. It is also shown that the initial failure strength or strain is greater in composites composed of low elongation and high elongation materials than in the pure low elongation composite. This is the result of the "size effect", that is the failure probability is lower in the composite with the smaller size of low elongation material. A good agreement between the theoretical and experimental results is achieved.

1. Introduction

An interesting phenomenon common to both unidirectional hybrid composites and $[\pm\theta/90]_s$ non-hybrid laminates exists. In unidirectional hybrid composites consisting of low elongation (LE) and high elongation (HE) fibres such as CFRP/GFRP, the failure strength (strain) of LE fibres under tension is often greater in the hybrid composite than in the pure LE fibre composite. This is known as a "hybrid effect" and has been extensively documented in the literature [1–10]. The increase in the failure strength (strain) of the LE fibres is more substantial for the smaller relative volume fraction of LE fibres and for the more dispersed fibre arrangements [6]. On the other hand, in the non-hybrid $[\pm\theta/90]_s$ $(0<\theta<90^\circ)$ laminates the failure strength (strain) of the 90° (LE) layer under tension is greater for the smaller thickness of the inner 90° layers [11–14]. It is also known that the failure strength (strain) of the inner 90° layer depends on the material properties of the outer $\pm\theta$ layers [13]. Thus, in a composite which consists of laminae with two different types of material properties such as in hybrid composites and $[\pm\theta/90]_s$ non-hybrid laminates, the failure strength (strain) of the LE layers is not on intrinsic material property and it depends on the adjacent HE material property and the geometrical arrangement.

This type of synergistic effect cannot be explained purely from the viewpoint of thermal residual stress or strain. It has been interpreted based upon both the energetic and statistical considerations. According to the energetic (fracture mechanics) approach, the hybrid effect results from the constraint to crack propagation by the HE material bridging a crack in the LE material [7]. This idea was adopted earlier by Parvizi *et al.* [11], who examined the constraint effect in non-hybrid cross-ply laminates in terms of the energetics of the fracture process. Parvizi *et al.* [11] note the conditions governing the initiation and

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growth of a crack remain unexplained, but nevertheless this approach has given interesting results in predicting crack constraint. Wang *et al.* [14] developed the fracture mechanics approach employing a finite element method and explained the mechanism of crack growth in the inner 90° layer of $[\pm 25/90]_s$ laminates.

Another approach to the problem of laminate composite strength is based upon the consideration of the statistical nature of the fibre and hence, composite strength. From the viewpoint of the statistical approach, a composite first fails at the weakest point and then, stress redistributions around a broken fibre or lamina occur and the next weakest point fails. It is very difficult to follow this failure process closely. To simplify the problem, the composite can be modeled as a chain of short fibre bundles in series, introducing the concept of ineffective length. Then the main task is reduced to the determination of the strength distribution function for short fibre bundles. The difficulty of this approach is in the analysis of the stress redistribution at local failures and the determination of all possible failure sequences. Harlow and Phoenix [15, 16] considered all sequences of fibre failures for a non-hybrid composite with a size less than 9 fibres by assuming a local load sharing rule. Fukuda and Chou [17, 18] analysed the failure strength and strain of hybrid composites using the solution of stress redistributions obtained from a shear-lag analysis. From a Monte Carlo simulation of the fibre failure, they concluded that the ultimate strength or strain of the hybrid composite was greater than that of the pure LE fibre composite, while the initial failure strengths or strains of the LE fibre composite and the hybrid composite were nearly the same. In a recent paper, Fukunaga *et al.* [19] showed that the initial failure strength of the LE fibres in an intermingled hybrid composite is also higher than that in a pure LE fibre composite, especially for low relative LE fibre volume fractions. They also showed that the multiple failure of LE and HE fibres would occur in the intermingled hybrid composite for high relative LE fibre volume fractions and for less uniform dispersion of the LE and HE fibres.

This paper deals with the initial failure strength of unidirectional hybrid laminates and $[\pm \theta/90]_s$ non-hybrid laminates where the LE laminae are sandwiched between HE laminae as shown in Fig. 1. Here, the term "initial failure" means failure of

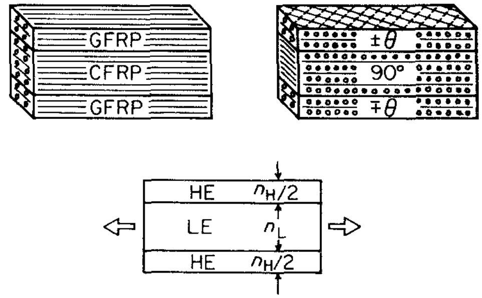

Figure 1 Analytical model for laminates. (a) GFRP/CFRP/GFRP laminate. (b) $[\pm \theta/90]_s$ laminate. (c) Model for analysis.

the n_L-layered LE composite. The simplest failure criterion for the n_L LE layers is that the first ply failure propagates through all LE layers instantly. Manders and Bader [7] showed that failure of the adjacent LE fibres of the order 3 resulted in the complete failure of the LE bundles from their experiment of hybrid composites. From this result, it may be a reasonable assumption that the first ply failure of LE layers propagates through all LE layers sandwiched between two neighbouring HE layers. Based upon the first ply failure criterion and the weakest link model, the initial failure strength and strain are predicted. As a special case, this prediction reduces to a simple volumetric relation [7, 13] which can be derived based on the assumption that the HE layers do not fail. The present theory is applied to the initial failure strength of unidirectional GFRP/CFRP/GFRP laminates and $[\pm \theta/90]_s$ CFRP laminates and good agreements between the analysis and the experimental data are obtained.

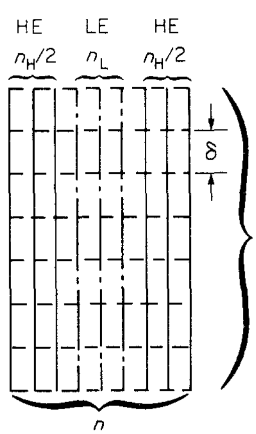

Figure 2 A chain-of-bundle model.

2. Strength analysis

2.1. First ply failure strength of HE/LE/HE laminates

In this section, the first ply failure strength of laminated composites under uniaxial tension is investigated. The laminates considered are the unidirectional hybrid laminates such as GFRP/CFRP/GFRP and $[\pm\theta/90]_s$ non-hybrid laminates. Both systems are modelled as combinations of high elongation (HE) and low elongation (LE) materials as shown in Fig. 1. Obviously, the CFRP of the GFRP/CFRP/GFRP unidirectional laminate and the 90° layer of the $[\pm\theta/90]_s$ laminate correspond to the LE component. The composite is modelled as a chain of short laminates in series as shown in Fig. 2. Each laminate has length δ equivalent to the ineffective length [5, 15]. Thus, the local length of the composite is $l = m\delta$. Also, each short laminate has n_L LE layers and n_H HE layers, where the total number of layers is $n = n_L + n_H$.

The aim of the analysis here is to obtain the first ply failure strength of the whole laminate composite from the strength characteristics of the LE and HE short layers. The cumulative distribution functions (c.d.f.) for the failure strain are assumed to follow the following two-parameter Weibull distribution functions for LE and HE short plies, respectively:

$$F_L(\epsilon) = 1 - \exp[-(\epsilon/\epsilon_L^*)^{a_L}]$$
$$F_H(\epsilon) = 1 - \exp[-(\epsilon/\epsilon_H^*)^{a_H}] \tag{1}$$

where ϵ_L^* and a_L, respectively, denote the scale and shape parameters for LE short layers, and ϵ_H^* and a_H, are the parameters for HE short layers. Then, the c.d.f. $H^c(\epsilon)$ for the first ply failure strain of the composite can be obtained by using the weakest link model as follows. First,

$$H^c(\epsilon) = 1 - [1 - G^c(\epsilon)]^m \tag{2}$$

where $G^c(\epsilon)$ denotes the c.d.f. for the first ply failure strain of the short laminate and is given by

$$G^c(\epsilon) = 1 - [1 - F_L(\epsilon)]^{n_L}[1 - F_H(\epsilon)]^{n_H} \tag{3}$$

Therefore,

$$H^c(\epsilon) = 1 - \exp[-mn_L(\epsilon/\epsilon_L^*)^{a_L} - mn_H(\epsilon/\epsilon_H^*)^{a_H}]. \tag{4}$$

It should be noted that this relation for the first ply failure strain is concerned with the composite laminate, not the LE layer because the failure of the LE layer does not always precede that of the HE layer. Similarly, the c.d.f. $H^L(\epsilon)$ for the first ply failure strain of the pure LE composite is given by

$$H^L(\epsilon) = 1 - \exp[-mn(\epsilon/\epsilon_L^*)^{a_L}]. \tag{5}$$

For a comparison of the failure strains for the HE/LE/HE laminate and the pure LE laminate, we consider the failure strains at 50% failure probability. From Equations 4 and 5, we obtain

$$\frac{n_L}{n}\left(\frac{\epsilon_c}{\epsilon_L^*}\right)^{a_L} + \frac{n_H}{n}\left(\frac{\epsilon_c}{\epsilon_H^*}\right)^{a_H} = \frac{\ln 2}{mn}$$
$$\left(\frac{\epsilon_L}{\epsilon_L^*}\right)^{a_L} = \frac{\ln 2}{mn} \tag{6}$$

where ϵ_c and ϵ_L, respectively, denote the median failure strain for the HE/LE/HE composite and the pure LE composite.

We are most interested in the relation between the ratio ϵ_c/ϵ_L of the median failure strains and the relative volume fraction of the LE material, n_L/n. It can be seen from Equation 6 that the ratio ϵ_c/ϵ_L varies with the composite size mn, the scale and shape parameters of the LE and HE materials as well as the volume fraction of the LE material, n_L/n. In the case of $a_L = a_H = a$, Equation 6 gives:

$$\frac{\epsilon_c}{\epsilon_L} = \left(\frac{n_L + n_H/R^a}{n}\right)^{-1/a} \tag{7}$$

where $R = \epsilon_H^*/\epsilon_L^*$. The ratio ϵ_c/ϵ_L in Equation 7 is independent of the composite size. By assuming linear stress–strain relations for LE and HE layers, the first ply failure strength ratio σ_c/σ_L of the HE/LE/HE laminate to the pure LE laminate is given by

$$\frac{\sigma_c}{\sigma_L} = \frac{\bar{E}}{\bar{E}_L}\left(\frac{n_L + n_H/R^a}{n}\right)^{-1/a} \tag{8}$$

where $\bar{E} = (n_L E_L + n_H E_H)/n$ denotes the average composite Young's modulus and E_L and E_H are, respectively, the Young's moduli of the LE and HE plies.

2.2. Determination of the scale and shape parameters of the angle plies

For the $[\pm\theta/90]_s$ laminate, the scale parameter ϵ_H^* and the shape parameter a_H of the $\pm\theta$ ply vary with the lamination angle θ. ϵ_H^* and a_H are now derived from the unidirectional laminate properties. The Young's modulus E_0 of an angle ply laminate is given by

$$1/E_0 = \cos^4\theta/E_1 + (1/G_{12} - 2\nu_1/E_1)\cos^2\theta \times \sin^2\theta + \sin^4\theta/E_2 \quad (10)$$

where E_1, E_2, ν_1 and G_{12} denote the longitudinal Young's modulus, transverse Young's modulus, the major Poisson's ratio and the in-plane shear modulus, respectively. The failure strength of an angle ply laminate is analysed by the following Tsai–Hill failure criterion:

$$(\sigma_1/\sigma_1^*)^2 - \sigma_1\sigma_2/\sigma_1^{*2} + (\sigma_2/\sigma_2^*)^2 + (\tau_{12}/\tau_{12}^*) = 1 \quad (11)$$

where

$$\sigma_1^* = E_1\epsilon_1^*, \sigma_2^* = E_2\epsilon_2^*, \tau_{12}^* = G_{12}\gamma_{12}^* \quad (12)$$

ϵ_1^*, ϵ_2^* and τ_{12}^* denote the longitudinal tensile, transverse tensile and the in-plane shear failure strains, respectively. These failure strains are random variables. Under the applied uniaxial tensile stress $\sigma_0 = E_0\epsilon_0$, the stresses shown in Equation 11 are [20]:

$$\sigma_1 = C_1\sigma_0, \ \sigma_2 = C_2\sigma_0, \ \tau_{12} = C_3\sigma_0 \quad (13)$$

where

$$\left.\begin{aligned} C_1 &= \cos^2\theta + 2\cos\theta\sin\theta\chi G \\ C_2 &= \sin^2\theta - 2\cos\theta\sin\theta\chi G \\ C_3 &= \cos\theta\sin\theta - (\cos^2\theta - \sin^2\theta)\chi G \\ \chi &= [\sin^2\theta/E_2 - \cos^2\theta/E_1 + (1/G_{12} - 2\nu_1/E_1) \\ &\quad \times(\cos^2\theta - \sin^2\theta)/2]\, 2\sin\theta\cos\theta \\ 1/G &= [(1+2\nu_1)/E_1 + 1/E_2]\, 4\cos^2\theta\sin^2\theta \\ &\quad + (\cos^2\theta - \sin^2\theta)^2/G_{12}. \end{aligned}\right\} \quad (14)$$

From Equations 11 and 13, we obtain

$$\sigma_0 = 1/[(C_1/\sigma_1^*)^2 - C_1C_2/\sigma_1^{*2} + (C_2/\sigma_2^*)^2 + (C_3/\tau_{12}^*)^2]^{1/2} \quad (15)$$

and

$$\epsilon_0 = 1 \Bigg/ \left[\left(\frac{E_0C_1}{E_1\epsilon_1^*}\right)^2 - \frac{E_0^2C_1C_2}{E_1^2\epsilon_1^{*2}} + \left(\frac{E_0C_2}{E_2\epsilon_2^*}\right)^2 + \left(\frac{E_0C_3}{G_{12}\gamma_{12}^*}\right)^2\right]^{1/2} \quad (16)$$

Thus, if each of the failure strains ϵ_1^*, ϵ_2^*, γ_{12}^* of a unidirectional lamina can be expressed by a two-parameter Weibull distribution function, the tensile failure strain ϵ_0 of an angle ply laminate is determined from Equation 16. Then, if it is assumed that the failure strain ϵ_0 can be expressed by a two-parameter Weibull distribution function, the corresponding scale and shape parameters are denoted by ϵ_H^* and a_H, respectively. This kind of problem was treated by Sun and Yamada [21] for the case of linear stress–strain relations and by Uemura and Fukunaga [22] for the case of non-linear stress–strain relations by using Monte Carlo simulation.

Let $\bar{\epsilon}_1^*$, $\bar{\epsilon}_2^*$ and $\bar{\gamma}_{12}^*$ be the scale parameters of ϵ_1^*, ϵ_2^* and γ_{12}^*, respectively, and a_1, a_2 and a_{12} the shape parameters of ϵ_1^*, ϵ_2^* and γ_{12}^*, respectively. Explicit solutions ϵ_H^* and a_H of Equation 16 are not readily available. For simplicity, we assume $a_1 = a_2 = a_{12} = a$. Then from Equation 16, we obtain

$$\epsilon_H^*/\bar{\epsilon}_2^* = 1 \Bigg/ \left[\left(\frac{E_0C_1\bar{\epsilon}_2^*}{E_1\bar{\epsilon}_1^*}\right)^2 - \frac{E_0^2C_1C_2\bar{\epsilon}_2^{*2}}{E_1^2\bar{\epsilon}_1^{*2}} + \left(\frac{E_0C_2}{E_2}\right)^2 + \left(\frac{E_0C_3\bar{\epsilon}_2^*}{G_{12}\bar{\gamma}_{12}^*}\right)^2\right]^{1/2}. \quad (17)$$

2.3. First ply failure and initial failure

In Sections 2.1 and 2.2, the discussions were concerned with the first ply failure of the laminate, not with the initial failure which means the failure of all the LE layers sandwiched in between two HE layers. The propagation of failure through all the LE layers after the first ply failure is determined by the strength characteristics of all the short plies and the stress redistributions due to ply failures. It is rather difficult to solve this problem exactly. Under certain simplification, such problems have been solved by the authors [17–19, 23]. In unidirectional composite, the first failure of a LE fibre does not result in complete failure of the composite, but the failure of a small group of about three fibres does induce composite failure [7]. From this result, it may be reasonable to assume that the first ply failure of a LE layer propagates through all LE layers. Furthermore, in the present problem of sandwiched laminates, the stress redistributions for both the HE/LE/HE laminate and the pure LE laminate, due to the first ply failure can be assumed to have similar magnitude. This assumption is valid especially when n_L is large. Thus, if the critical size at which the unstable crack propagates through all LE layers, is smaller than the thickness of n_L LE layers, the ratio of the initial failure strength $(\sigma_c)_{max}/(\sigma_L)_{max}$ would be equal to the ratio of the first ply failure

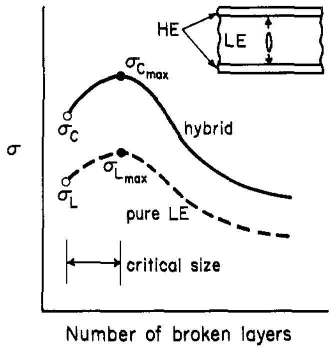

Figure 3 First ply failure and initial failure.

strength σ_c/σ_L as shown in Fig. 3 [14]. Although in Section 3 the analytical treatment is for first ply failure strength, the present approach can be applied to predict the *initial* failure strength when the focus is on obtaining $(\sigma_c/\sigma_L)_{\text{initial failure}}$ from $(\sigma_c/\sigma_L)_{\text{first failure}}$. The composite behaviour after initial failure is dominated by the stress redistribution as the crack extends [17–19, 23, 24].

3. Numerical results and discussion

In order to examine the characteristics of the initial failure strain, examples of the first failure strain ratio ϵ_c/ϵ_L at 50% failure probability are shown in Fig. 4. Here, the volume size $mn = 10^3$ and the shape parameter of LE layer $a_L = 5$ are used. For a_H/a_L less than unity, the first ply failure strain, ϵ_c, of the HE/LE/HE laminate becomes smaller than that of the pure LE laminate ϵ_L, even if the ratio of the failure strains of the short layers $\epsilon_H^*/\epsilon_L^*$ is greater than unity. For $a_H/a_L = 1$, ϵ_c/ϵ_L increases with the HE layer volume fraction as shown in Equation 7. For $a_H/a_L > 1$, the increase of ϵ_c/ϵ_L with HE layer volume fraction is more remarkable than that for $a_H/a_L = 1$. In the case of $\epsilon_H^*/\epsilon_L^* \gg 1$ and $a_H/a_L \gg 1$, the contribution of the term $(\epsilon_c/\epsilon_H^*)^{a_H}$ in Equation 6 is negligible as compared with $(\epsilon_c/\epsilon_L^*)^{a_L}$ except in the neighbourhood of $n_L/n = 0$. Then we obtain

$$\epsilon_c/\epsilon_L \cong (n/n_L)^{1/a_L}. \qquad (18)$$

This equation represents a simple volumetric effect and was applied by Manders and Bader [7] for hybrid composites and by Flaggs and Kural [13] for the $[\pm\theta/90]_s$ laminates. The curve for $a_H = 10$ and $\epsilon_H^*/\epsilon_L^* = 2.0$ is almost the same as that shown in Equation 18.

Next, we consider the case of GFRP/CFRP/GFRP unidirectional hybrid composite. By assuming the same shape parameter $a_L = a_H = a$ for CFRP and GFRP, we can apply Equation 7. For $\epsilon_H^*/\epsilon_L^* = 3.0$ and $E_H/E_L = 1/3$, the ratios of the first ply failure strain ϵ_c/ϵ_L and the first ply failure strength σ_c/σ_L are, respectively, shown in Figs. 5 and 6. In Fig. 6, points A and D, respectively, represent the strength of GFRP and CFRP composite. The line BD ($\sigma_c/\sigma_L = \bar{E}/E_L$) represents the stress in the hybrid at which failure of CFRP takes place. The line AE ($\sigma_c/\sigma_L = n_H/n_L + n_H$) represents the stress in the hybrid assuming that CFRP carries no load. Fig. 7 shows comparison of the present theory with the experimental results of Manders and Bader [6] for HTS carbon/E-glass hybrid laminates where the effect of thermal strain

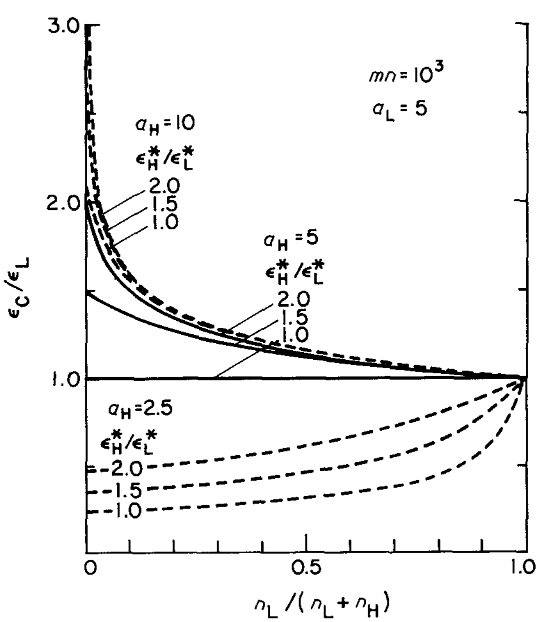

Figure 4 The relation between initial composite failure strain ϵ_c (normalized by ϵ_L) and relative LE-layer volume fraction.

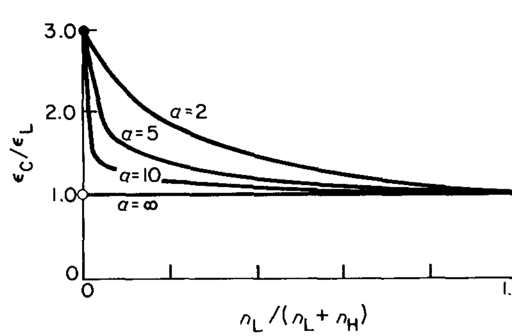

Figure 5 Initial composite failure strain ϵ_c (normalized by ϵ_L) in GFRP/CFRP/GFRP laminates. $a_L = a_H = a$, $\epsilon_H^*/\epsilon_L^* = 3.0$ and $E_H/E_L = 1/3$.

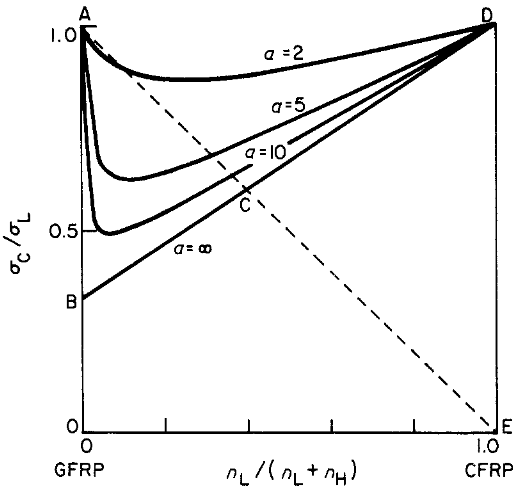

Figure 6 Initial composite failure strength σ_c (normalized by σ_L) in GFRP/CFRP/GFRP laminates. $a_L = a_H = a$, $\epsilon_H^*/\epsilon_L^* = 3.0$ and $E_H/E_L = 1/3$.

is taken into consideration. For failure strains, $\epsilon_L^* = 0.0115$ and $\epsilon_H^* = 0.0280$ are used [6]. Analytical values from Equation 7 are shown for $a = 5$, 10 and 20. The analytical results for $a = 10$ show good agreement with the experimental results. As the shape parameter a decreases, the first ply failure strain and strength of the hybrid composite increase relative to those of the pure CFRP composite. When $a \to \infty$, that is, for composites without scattering in the LE and HE layer failure strains, the first ply failure strain of the hybrid composite is identical to that of the pure CFRP composite. This relation at $a \to \infty$ is known as the rule of mixtures for the initial failure in hybrids [6, 18]. The enhancement of the failure strain above the level predicted by the rule of mixtures is known as a "hybrid effect". The hybrid effect is greater for smaller values of the shape parameter and for smaller value of the LE-layer volume fraction.

Next, we consider the case of $[\pm\theta/90]_s$ CFRP laminate. Experimental values of the elastic lamina material properties obtained by Flaggs and Kural [13] are adopted here:

$$E_1 = 20 \times 10^6 \text{ psi}, \; E_2 = 1.7 \times 10^6 \text{ psi},$$
$$G_{12} = 0.66 \times 10^6 \text{ psi}, \; \nu_1 = 0.29. \qquad (19)$$

Strength properties for a CFRP unidirectional composite are assumed as follows:

$$\frac{E_1\bar{\epsilon}_1^*}{E_2\bar{\epsilon}_2^*} = 30, \quad \frac{G_{12}\bar{\gamma}_{12}^*}{E_2\bar{\epsilon}_2^*} = 1.6. \qquad (20)$$

It is also assumed that $a_H = a_1 = a_2 = a_{12}$. Therefore, Equation 7 can be used to obtain the ratio of the first ply failure strains ϵ_c/ϵ_L. Fig. 8 shows the relations between ϵ_c/ϵ_L and the 90° layer volume fraction. It can be seen from this figure that the first ply failure strain of the 90° layer depends upon not only the 90° layer volume fraction $n_L/(n_L + n_H)$ and the shape parameter but also the adjacent HE-layer ($\pm\theta$ laminate) failure strain. The HE-layer failure strains for $\theta = 0°$, 30° and 60° are, respectively, $\epsilon_H^* = 2.550\,\epsilon_L^*$,

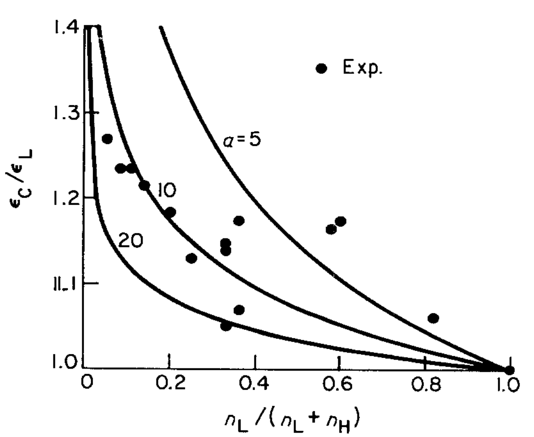

Figure 7 Comparisons of the present theory (Equation 7) with experimental results of Manders and Bader [6] for HTS carbon/E-glass hybrid laminates.

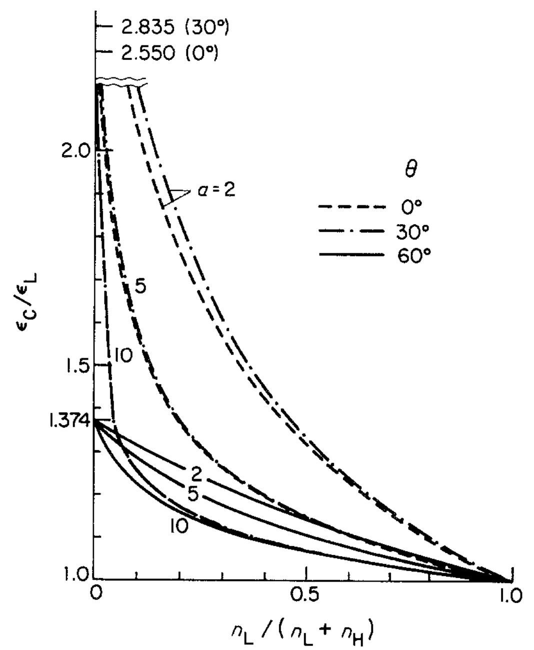

Figure 8 The relation between initial composite failure strain ϵ_c (normalized by ϵ_L) and relative 90°-layer volume fraction in $[\pm\theta/90]_s$ laminates, for $a_H = a_1 = a_2 = a_{12}$. (Curves for $\theta = 0°$ and 30° overlap for $a = 10$.)

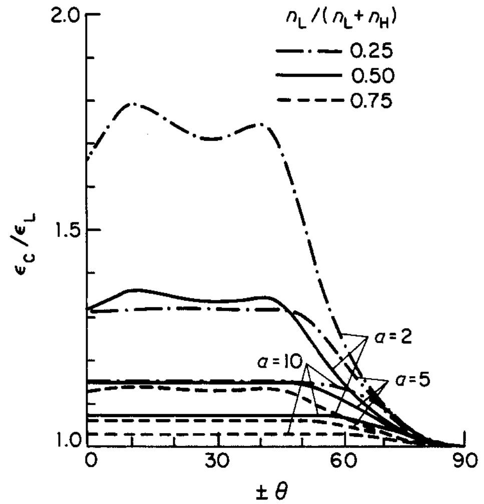

Figure 9 The relation between initial failure strain ϵ_c (normalized by ϵ_L) and the lamination angle of the angle plies in $[\pm\theta/90]_s$ laminates.

$2.835\epsilon_L^*$ and $1.374\epsilon_L^*$. The HE-layer failure strain ϵ_H^* for $\theta = 60°$ is not much larger than that of the 90° layer ϵ_L^*. Therefore, for $\theta = 60°$ we cannot use a simple volumetric relation shown in Equation 18 to predict the first ply failure strain, while for $\theta = 0°$ and 30°, Equation 7 is almost the same as Equation 18 except in the neighbourhood of $n_L/n = 0$. Fig. 9 shows the relation between the ratio ϵ_c/ϵ_L and the lamination angle θ. For θ less than 45°, the enhancement in the first ply failure strain of the $[\pm\theta/90]_s$ laminate is substantial but for θ larger than 45°, ϵ_c/ϵ_L drops very rapidly with the increase in θ. Fig. 10 shows the com-

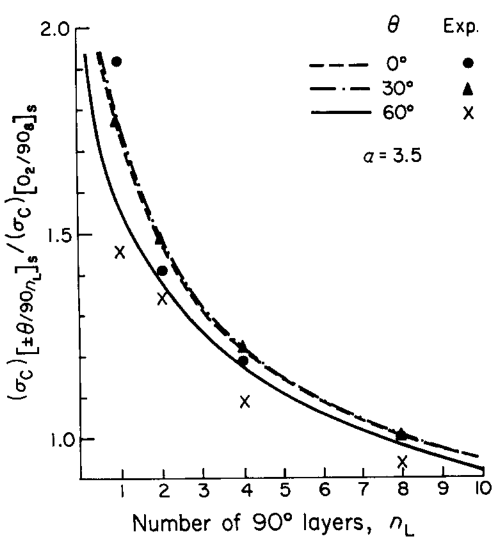

Figure 10 Comparisons of the present theory with experimental results of Flaggs and Kural [13]. $a = a_H = a_L = 3.5$.

parison of the initial failure strength between the present theory and experimental results of Flaggs and Kural [13]. The effect of the thermal residue stress has been taken into consideration. In this figure, the initial composite failure strength is normalized by the strength of the 90° layer in the $[\theta_2/90_8]_s$ laminate and the shape parameter of $a = 3.5$ is assumed. A rather good agreement between the theory and experiment is demonstrated for $\theta = 0°$ and 30°. For $\theta = 60°$, the theoretical results are slightly higher than the experimental values. This discrepancy may be attributed to the assumptions regarding the shape parameters ($a_L = a_H$) and the linear stress–strain relation of the $\pm\theta$ angle-ply laminates.

4. Conclusions

The initial failure strength and strain at the failure of low elongation layers in hybrid and non-hybrid laminates have been analysed by a statistical approach based upon the weakest link model. First ply failure is adopted as the criterion for the initial failure of the composites. The formula for determining the first ply failure strength is derived. The following conclusions are pertinent.

1. The initial failure strain of the LE layers is given by Equation 6 by taking the effect of the adjacent HE layers into consideration. For the special case of $a_L = a_H = a$, Equation 6 is reduced to Equation 7.

2. The initial failure strain of HE/LE/HE composites varies with the composite size, the scale and shape parameters of the LE and HE materials as well as the volume fraction of the LE material. In the case of $a_L = a_H$, the composite initial failure strain increases relative to that of the pure LE composite as the shape parameter and the relative LE fibre volume fraction decrease.

3. The failure strain of the 90° ply in $[\pm\theta/90]_s$ laminates depends not only upon the 90° layer volume fraction but also the adjacent HE-layer failure strain.

4. Comparisons between the present analytical results and the existing experimental data for both unidirectional GFRP/CFRP/GFRP laminates and $[\pm\theta/90]_s$ CFRP laminates have shown good agreement.

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